High View Primary School

Calculation Policy

Contents

Introduction

Key aims of the national curriculum for mathematics aims to ensure that all pupils:

• become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.

• **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language

• can **solve problems** by applying their mathematics to a variety of routine and non routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

This booklet shows teachers and parents the strategies we teach children when teaching the four operations.

The four operations are addition, subtraction, multiplication and division. Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

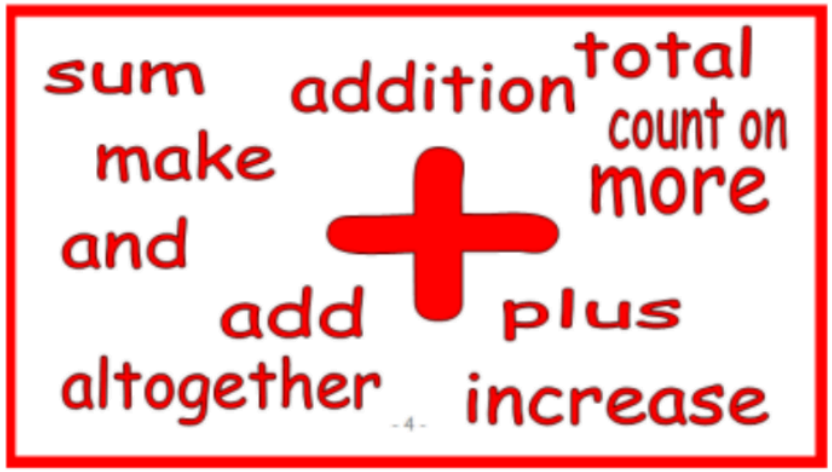
Strategies for calculation need to be supported by familiar models, images and most importantly *accurate vocabulary* to reinforce understanding. When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the concept.

The transition between stages *should not* be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately. We will follow a Mastery style approach to Maths, where children explore concepts using concrete manipulatives, move into creating pictorial representations and finally move into more abstract written calculations.

In this booklet you will find we have ordered of the types of strategies your child will encounter. We need to emphasise that children will always be extended or indeed supported whatever year group they are in.

Addition



Manipulatives, Models and Images

Counting apparatus

Numicon

Tens frame

Whole part diagrams

Base 10

Cusinaire

Bar Model

Place value cards

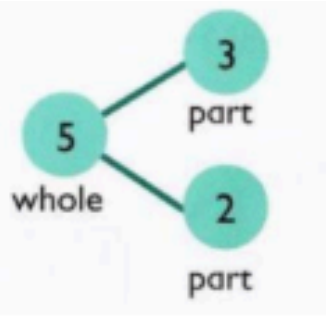
Counting stick

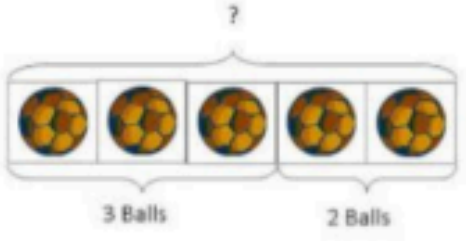
Bead string

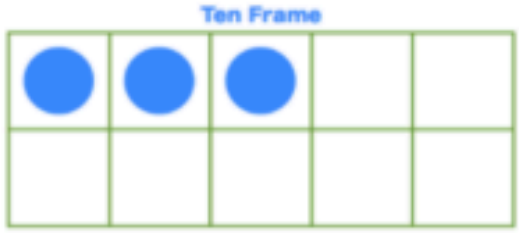
Number tracks

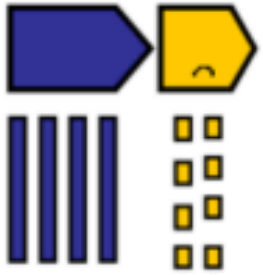
Number lines

Hundred square







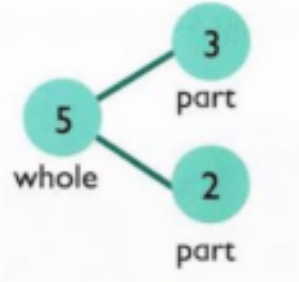


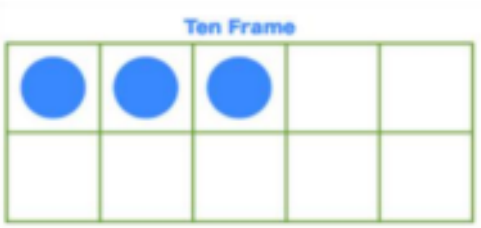
Subtraction

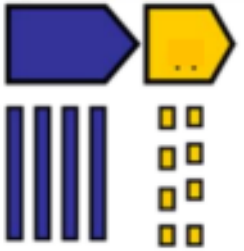
subtract
count on
count back
fewer
less
minus
take away
finding the difference

Manipulatives, Models and Images

- Counting apparatus
- Numicon
- Tens frame
- Whole part diagrams
- Base 10
- Cusinaire
- Bar Model
- Place value cards
- Counting stick
- Bead string
- Number tracks
- Number lines
- Hundred square



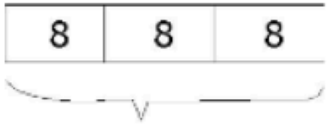


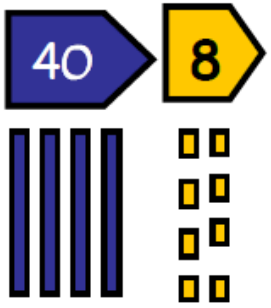


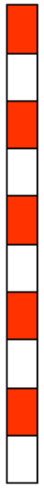
Multiplication

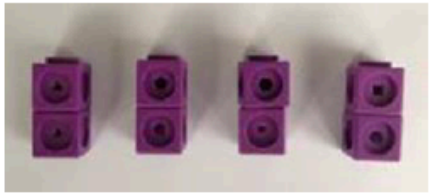
Manipulatives, Models and Images

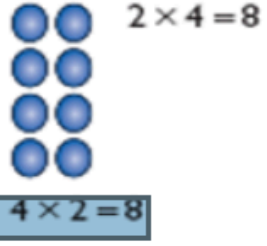
- Numicon
- Base 10
- Arrays
- Number tracks
- Numbered number line
- Multiplication squares
- Counting stick
- Bead strings

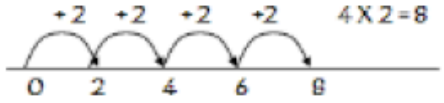






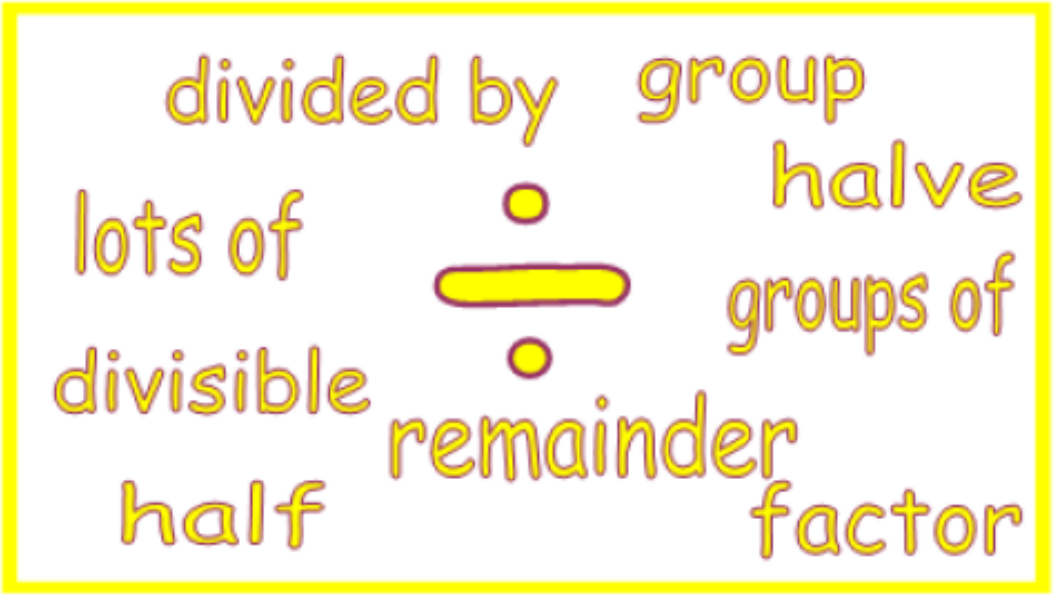


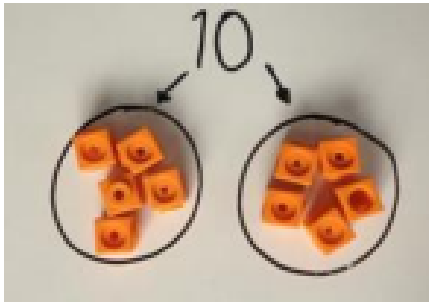


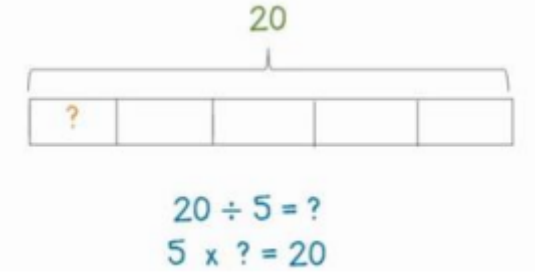


Division

Manipulatives, Models and Images
Counting apparatus
Arrays
Bar Model
100 squares
Number tracks
Numbered number lines
Empty number lines
Multiplication squares







Number: EYFS

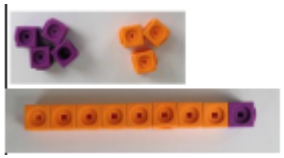
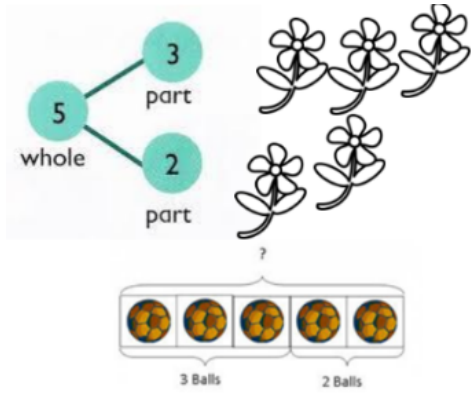
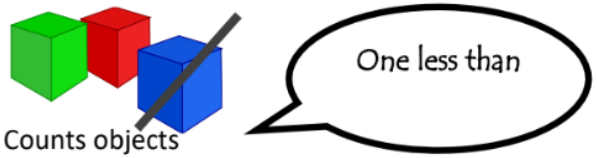
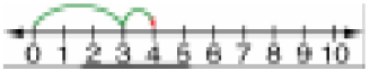
Objectives related to number, place value, addition and subtraction:

Nursery	**Reception**
Develop fast recognition of up to 3 objects, without having to count them individually (‘subitising’). Recite numbers past 5. Say one number for each item in order: 1,2,3,4,5. Know that the last number reached when counting a small set of objects tells you how many there are in total (‘cardinal principle’). Show ‘finger numbers’ up to 5. Link numerals and amounts: for example, showing the right number of objects to match the numeral, up to 5. Experiment with their own symbols and marks as well as numerals. Solve real world mathematical problems with numbers up to 5. Compare quantities using language: ‘more than’, ‘fewer than’.	Count objects, actions and sounds. Subitise. Link the number symbol (numeral) with its cardinal number value. Count beyond ten. Compare numbers. Understand the ‘one more than/one less than’ relationship between consecutive numbers. Explore the composition of numbers to 10. Automatically recall number bonds for numbers 0–5 and some to 10.

Addition and Subtraction links

	Concrete	Pictorial	Abstract
Counts up to three or four objects by saying one number name for each item. (same for up to and beyond 10)	Real objects 	Child draws 	Child can attach numerals to their objects and pictures  1 2 3 4
Counts an irregular arrangement of up to ten objects.		Child draws 	7 4
Says the number that is one more than a given number.		Child draws 	

Represents numbers in different ways	 Child is able to represent 5 using fingers, cubes, counters on a tens frame and lastly as a numeral.	 Child is able to mark make in a variety of ways to represent numbers.	Child is able to recognise and write the numeral 5.

Finds the total number of items in two groups by counting all of them.	 Use cubes to add two numbers together as a group or in a bar.	 Use pictures to add two numbers together as a group or in a bar.	3 + 5 = 8 5 + 3 = 8 Use the part-part whole diagram as shown above to move into the abstract. 
Finds one more or one less from a group of up to five objects, then ten objects.			Can complete the following: *One less than 4 is* ____ *One more than 4 is* _____
Counts on or back to find the answer	Physically moves along a number track. 		

Key Stage 1

Mental skills

Addition	Subtraction
Recognise the value and position of numbers Count on in ones and tens Know number bonds to 10 and 20 Add multiples of 10 to any number Partition and recombine numbers Bridge through 10 Use inverse operations to check answers (3+4=7 4+3=7 7-3=4 7-4=3)	Recognise the value and position of numbers Count on or back in ones and tens Know number facts for all numbers to 20 Subtract multiples of 10 from any number Partition and recombine numbers Bridge through 10 Using inverse operation to check answer (e.g. 5+6=11 11-6 = 5 11-5=6 11-6 = 5)
Stem sentences and generalisations - The biggest number will always be the total. 10 + 3 = 13 - Each side of the equal sign must total the same amount. 5 + 5 = 10 - The total is the amount altogether. 3 + 4 = 7 - If you add the amounts in different orders you will always get the same total 2+ 8 = 10 8 + 2 = 10	**Stem sentences and generalisations** - The biggest number always has to come first. 10 - 3 = 7 - Each side of the equal sign must total the same amount. 15 - 5 = 10 - The total is the amount altogether. 3 + 4 = 7 - You will always (unless taking away 0) end up with less than you started with.

Addition

Progression of calculation

O + O (aggregation: combining objects)

1 more than a given number up to 10.

O + O (augmentation: counting on)

Subtraction

Progression of calculation

O – O (take away structure)

1 less than a given number: First from 1 – 5, moving to 1 – 10 and finally 1- 20

O – O (comparison structure, “how many more …; “how many less …”)

Addition and Subtraction: Year 1

Objectives related to number, place value, addition and subtraction:

Year One National Curriculum objectives

Number and Place Value

- **count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number • count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens**
- **given a number, identify one more and one less**
- **identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least**
- **read and write numbers from 1 to 20 in numerals and words.**

Addition and Subtraction

- **read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs • represent and use number bonds and related subtraction facts within 20**
- **add and subtract one-digit and two-digit numbers to 20, including zero**
- **solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ? – 9**

Addition and Subtraction: Year 2

Objectives related to number, place value, addition and subtraction

Year Two National Curriculum objectives

Number and Place Value

- **count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward**
- **recognise the place value of each digit in a two-digit number (tens, ones)**
- **identify, represent and estimate numbers using different representations, including the number line • compare and order numbers from 0 up to 100; use <, > and = signs**
- **read and write numbers to at least 100 in numerals and in words**
- **use place value and number facts to solve problems.**

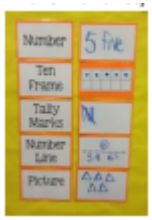
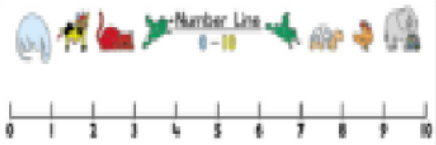
Addition and Subtraction

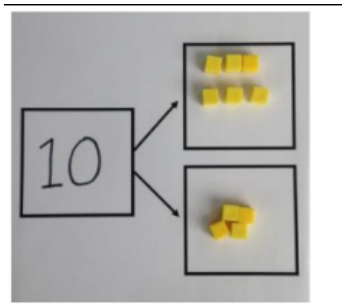
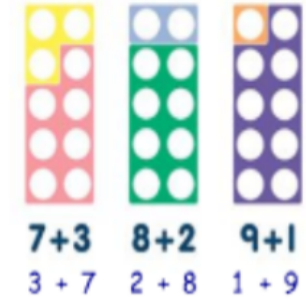
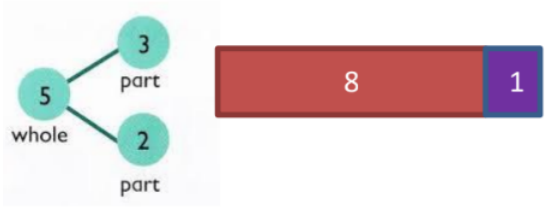
- **solve problems with addition and subtraction:**
- **using concrete objects and pictorial representations, including those involving numbers, quantities and measures**
- **applying their increasing knowledge of mental and written methods**

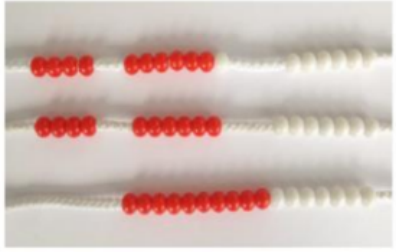
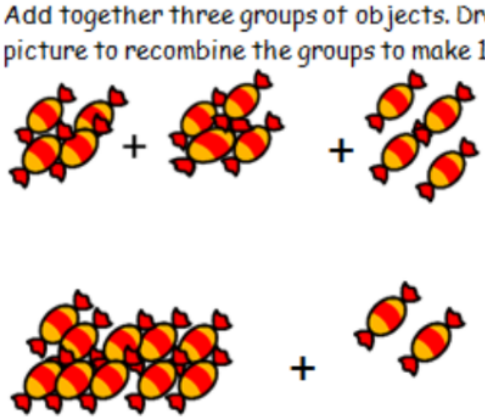
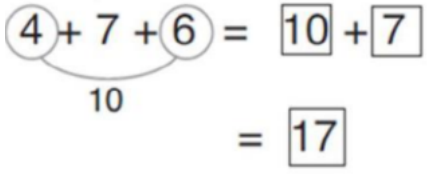
- **recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100**
- **add and subtract numbers using concrete objects, pictorial representations, and mentally, including:**
- **a two-digit number and ones**
- **a two-digit number and tens**
- **two two-digit numbers**
- **adding three one-digit numbers**
- **show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.**

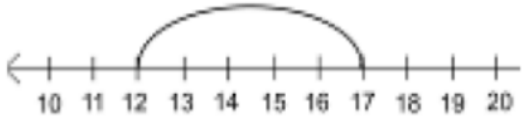
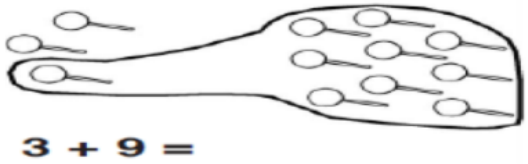
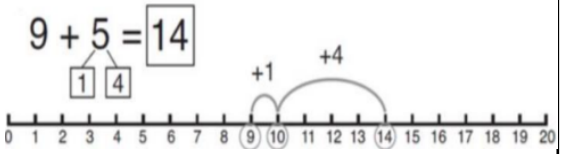
Key Stage 1

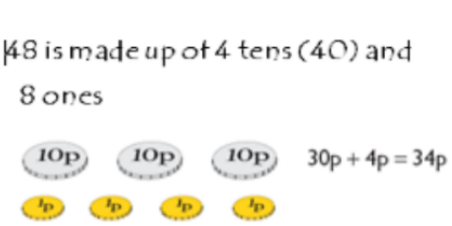
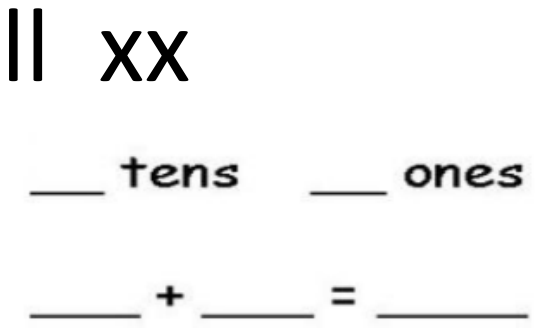
Addition

	Concrete	Pictorial	Abstract
Identify and represent numbers using objects and pictorial representations including the number line	 Child is able to represent 5 using fingers, cubes, counters on a tens frame and lastly as a numeral.	 Child is able to mark make in a variety of ways to represent numbers.	Child is able to recognise and write the numeral 5 and find it on a number line. 

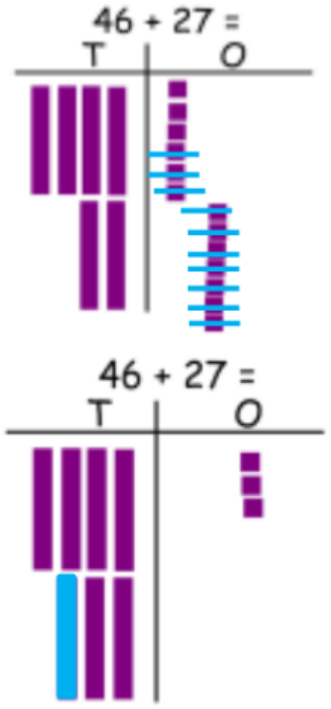
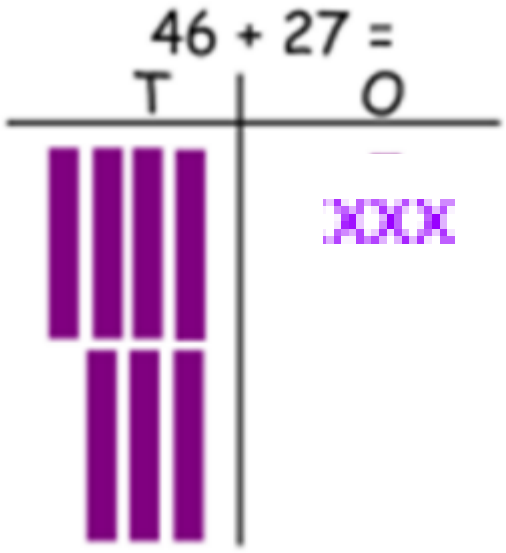
Know that addition can be done in any order	 		5 = 3 + 2 5 = 2 + 3 9 = 8 + 1 9 = 1 + 8

Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.		Combine the two numbers that make 10 and then add on the remainder. 

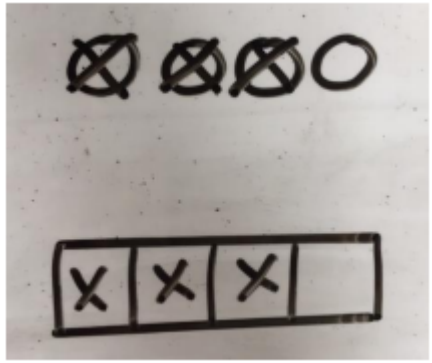
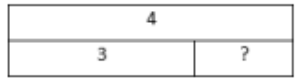
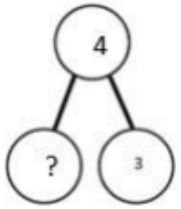
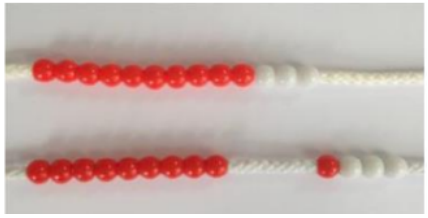
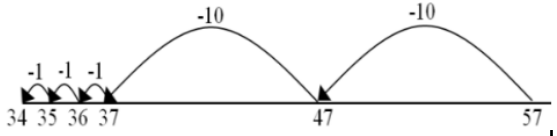
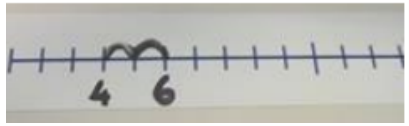
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	 12 + 5 = 17 Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10.	9 + 3 = 12 (9 + 1 + 2 = 12)   6 + 7 = 13 Start with the first number and use the next number to make 10 and a bit.	 Use pictures or a number line. Regroup or partition the smaller number to make 10.  	7 + 4= 11 (7 + 3 + 1 = 11) If I am at seven, how many more do I need to make 10? How many more do I add on now?

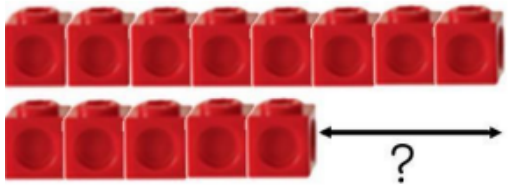
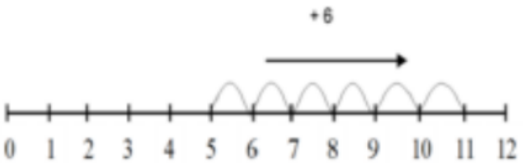
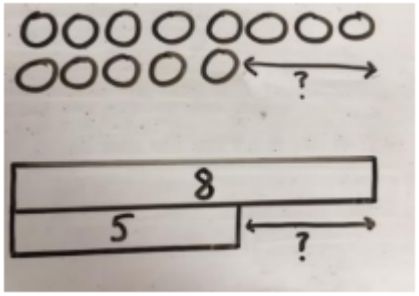
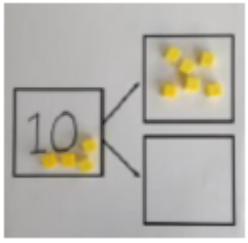
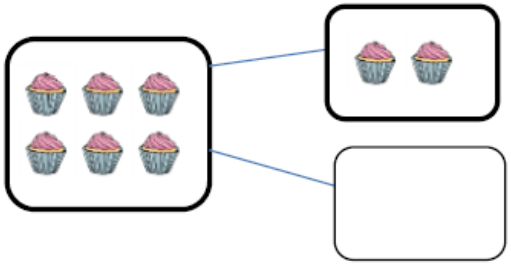
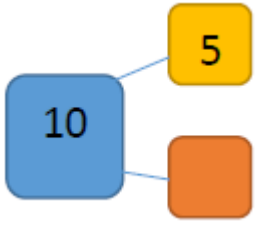
Understand how to partition. Knowing how to represent any number to 100 using manipulatives, drawings or jottings	Practically using arrow cards and base 10 to represent numbers   	 	20 + 7 = 27 10 + 10 + 7 = 27

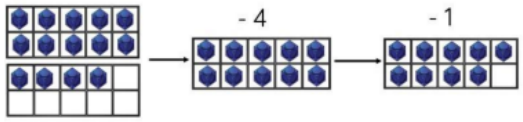
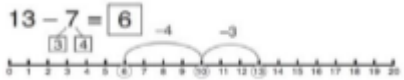
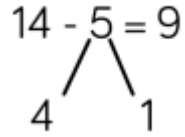
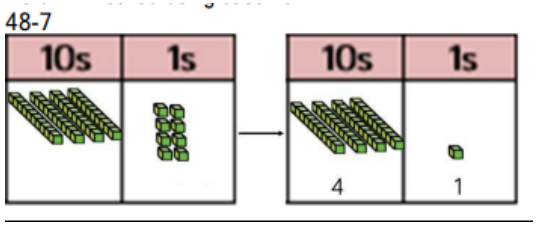
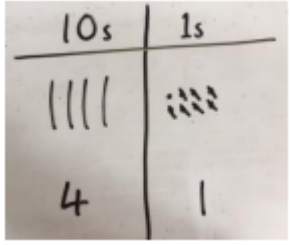
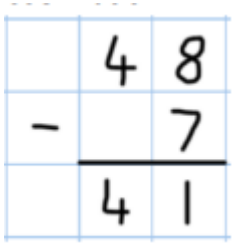
Column method- no regrouping	24 + 15= 39 Add together the ones first then add the tens. Use the base 10. 	After practically using the base 10, children can draw the base 10 to help them to solve additions. 	21 + 42 = 21 + 42

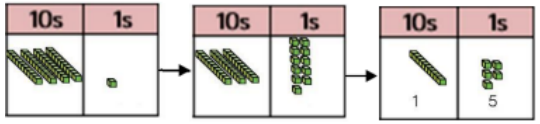
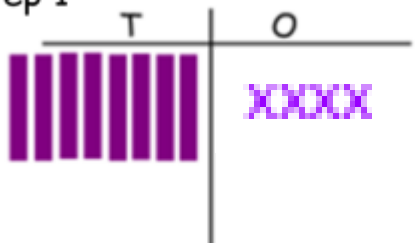
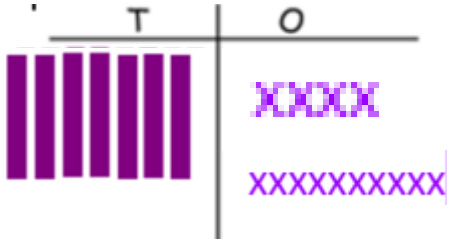
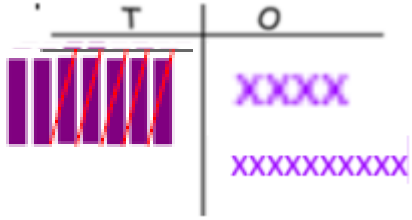
Column method- regrouping	**Make** both numbers on a place value grid.  Add up the ones and exchange 10 ones for one 10.	Children can **draw** a pictorial representation of the columns and place value counters to further support their learning and understanding.  	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 46 + 27 = 40 + 6 20 + 7 60 + 13 = 73

Subtraction

	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away. 4- 2 = 2  (ten frames, Numicon, cubes and other items such as beanbags could be used)	Cross out drawn objects to show what has been taken away. 	4- 3 = ☐ = 4 – 3  
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4  Use counters or cubes and move them away from the group as you take them away counting backwards as you go. 6 - 2 = 4 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	Put 6 in your head, count back 2. What number are you at? Use your fingers to help. (Encourage children to use an empty number line) 

Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference Use basic bar models with items to find the difference.	Count on to find the difference.  Draw bars to find the difference between 2 numbers.  	Find the difference between 8 and 5. 8 – 5, the difference is ___ Children to explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference
Part Whole Model to support subtraction and inverse	Link to addition- use the part whole model to help explain the inverse between addition and subtraction.  If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part whole model. 	Move to using numbers within the part whole model. 

Make 10 (Partitioning numbers in order to subtract)	14 – 5 = (14 – 4 – 1 =)  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	Children to show how they can make 10 by partitioning the subtrahend.  14 – 4 = 10 10 – 1 = 9 How many do we take off to reach the next 10? How many do we have left to take off?
Column method without regrouping	Use Base 10 to make the bigger number then take the smaller number away. 	Draw the base 10 alongside the written calculation to help to show working.  48-7	This will lead to a clear written column subtraction. 

Column method with regrouping (More able year 2)	Make the larger number with the base 10 or counters 41 - 26  	84-56 Step 1  Step 2  70 14  20 8 Draw the base 10 onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.	84 - 56 = 4 - 6 = can't do it, I need to exchange 80 -10 = 70 4 + 10 = 14 14 - 6 = 8 70 - 50 = 20 20 + 8 = 28

Addition and Subtraction: Year 3

Objectives related to addition and subtraction

Year Three National Curriculum objectives:

- add and subtract numbers mentally, including:
  - a three-digit number and 1s
  - a three-digit number and 10s
  - a three-digit number and 100s
- add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

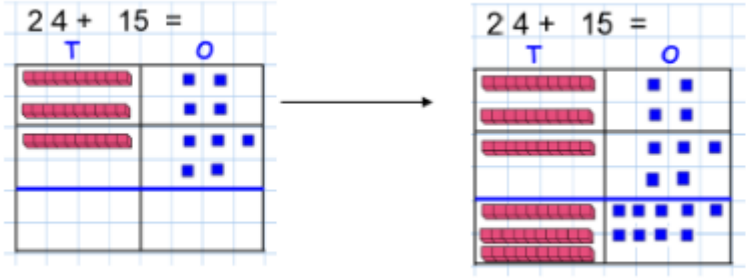
Mental Objectives

Recall	Mental Calculation Skills	Mental Strategies
•addition and subtraction facts for all numbers up to at least 10, e.g. 3 + 4, 8 – 5 • number pairs with totals to 20 • all pairs of multiples of 10 with totals up to 100, e.g. 30 + 70, or 60 +□= 100 • what must be added to any two-digit number to make the next multiple of 10, e.g. 52 +□= 60 • addition doubles for all numbers to 20, e.g. 17 + 17 and multiples of 10 to 50, e.g. 40 + 40	• add or subtract a pair of single-digit numbers, including crossing 10, e.g. 5 + 8, 12 – 7 • add any single-digit number to or from a multiple of 10, e.g. 60 + 5 • subtract any single-digit number from a multiple of 10, e.g. 80 – 7 • add or subtract a single-digit number to or from a two-digit number, including crossing the tens boundary, e.g. 23 + 5, 57 – 3, then 28 + 5, 52 – 7 • add or subtract a multiple of 10 to or from any two-digit number, e.g. 27 + 60, 72 – 50 • add 9, 19, 29, … or 11, 21,31, … • add near doubles, e.g. 13 + 14, 39 + 40	• reorder numbers when adding • count on in fours, eights, fifties and hundreds. • partition: bridge through 10 and multiples of 10 when adding and subtracting • partition and combine multiples of tens and ones • use knowledge of pairs making 10 • partition: count on in tens and ones to find the total • partition: count on or back in tens and ones to find the difference • partition: add a multiple of 10 and adjust by 1 • partition: double and adjust

Addition

Progression of Calculation

TO + TO (bridging through 10s, not crossing 100)
TO + TO (crossing 100)
HTO + O
HT0 + TO
HT0 + HTO

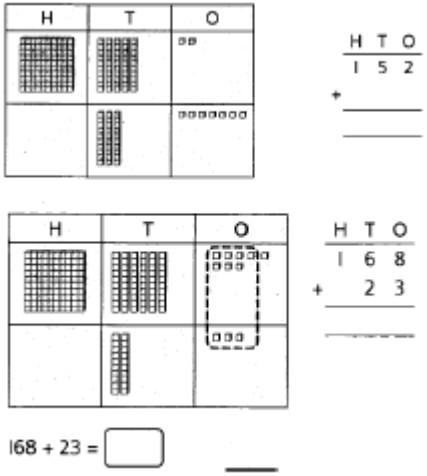


	T	O
	2	4
+	1	5
	3	9

Using the base 10 children are encouraged to create the values and then write the written method next to their addition chart.

Children can draw the pictorial representation of the columns to further support their learning and understanding.

When adding 2 and 3 digit numbers, children should be allowed the opportunity to record their own jottings to support their understanding.



E.g.

46 + 87 = 87 + 40 + 6

Or

40 + 80 = 120

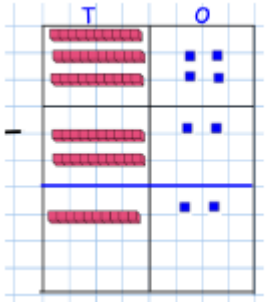
6 + 7 = 13

Progression of Calculation

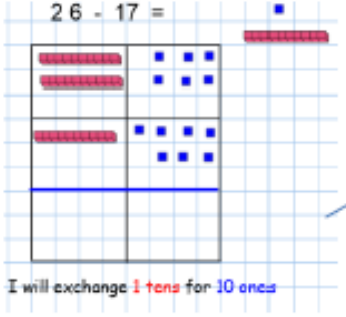
TO – TO

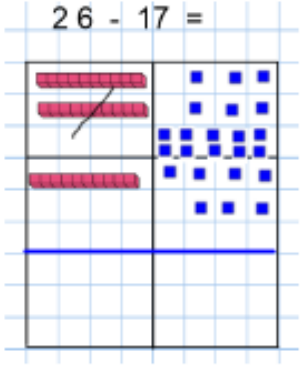
HTO -TO

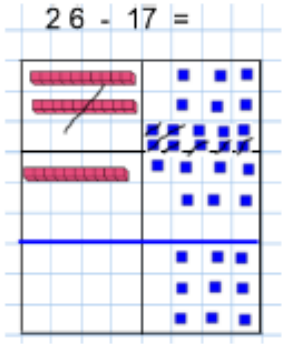
HTO - HTO



	T	O
	3	4
-	2	2
	1	2







Using the base 10 children are encouraged to create the values and then write the written method next to their addition chart.

Children can draw the pictorial representation of the columns to further support their learning and understanding.

When adding 2 and 3 digit numbers, children should be allowed the opportunity to record their own jottings to support their understanding.

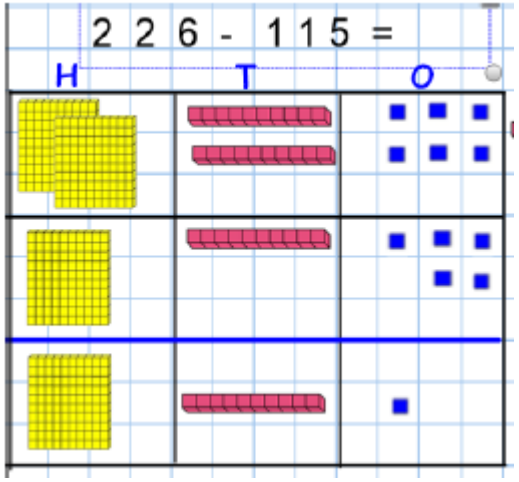
E.g.

46 + 87 = 87 + 40 + 6

Or

40 + 80 = 120

6 + 7 = 13



Addition and Subtraction: Year 4

Objectives related to addition and subtraction

Year Four National Curriculum objectives:

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why

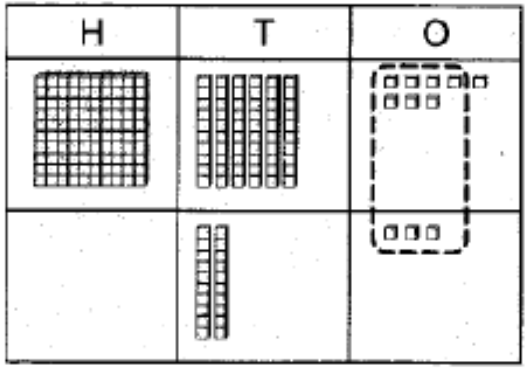
Recall	Mental Calculation Skills	Mental Strategies
• sums and differences of pairs of multiples of 10, 100 or 1000 • addition doubles of numbers 1 to 100, e.g. 38 + 38, and the corresponding halves • what must be added to any three-digit number to make the next multiple of 100, e.g. 521 +□= 600 • pairs of fractions that total 1	• add or subtract any pair of two-digit numbers, including crossing the tens and 100 boundary, e.g. 47 + 58, 91 – 35 • add or subtract a near multiple of 10, e.g. 56 + 29, 86 – 38 • add near doubles of two-digit numbers, e.g. 38 + 37 • add or subtract two-digit or three-digit multiples of 10, e.g. 120 – 40, 140 + 150	• count on or back in thousands, hundreds, tens and ones • count on and backwards in sixes, sevens, nines, twenty-fives and thousands. • partition: add tens and ones separately, then recombine • partition: subtract tens and then ones, e.g. subtracting 27 by subtracting 20 then 7 • subtract by counting up from the smaller to the larger number • partition: add or subtract a multiple of 10 and adjust, e.g. 56 + 29 = 56 + 30 – 1, or 86 – 38 = 86 – 40 + 2 • partition: double and adjust • use knowledge of place value and related calculations, e.g. work out 140 + 150 = 290 using 14 + 15 = 29 • partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times)

Addition

Progression of Calculation

HTO + HTO (crossing 1000)
ThHTO + TO
ThHTO + HTO
ThHTO + ThHTO
£O.t h + £O.t h
£TO.t h + £TO.t h

Written methods

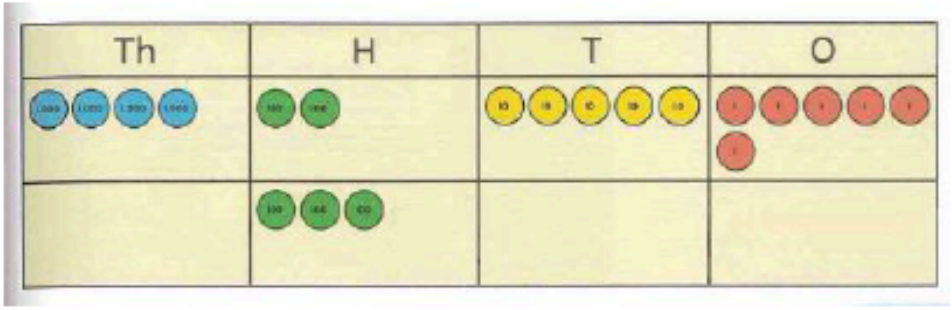


	H	T	O
	1	6	8
+		2	3

Once children are confident with the base 10 and have a solid understanding they move on to use place value counters to assist their calculations.

It is important that children use the manipulatives as it helps to embed their understanding of place value and using the written method alongside it will help them to understand the exact steps they are taking.

When it comes to adding money the place value counters include: 0.1, 0.01 and the place value charts for adding and subtracting include the decimal point.



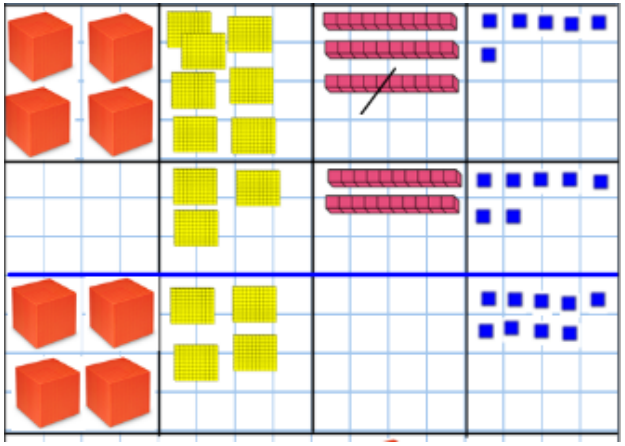
Th	H	T	O
4	2	5	6
+	3	0	0
4	5	5	6

Subtraction

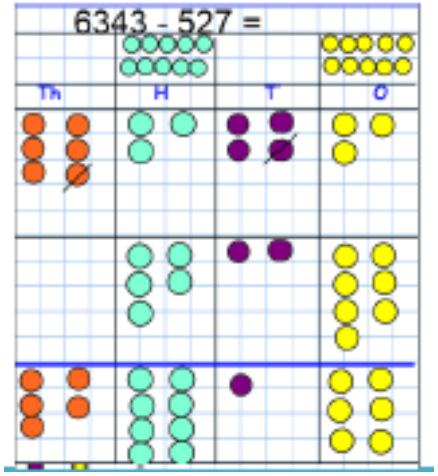
Progression of Calculation

HTO – TO
HTO – HTO
ThHTO - HTO
Stage 4: Exchange of T to O and H to T
Stage 5: Handling of zero place holders, e.g. £O. t h - £O. t h

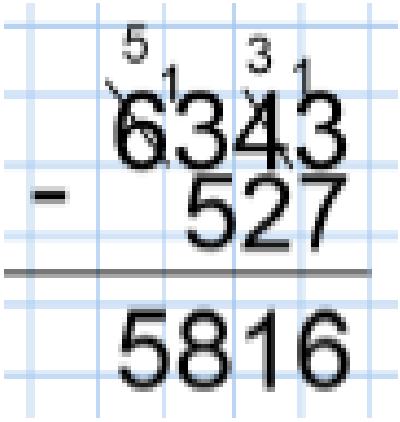
Concrete
(Manipulating Base 10 and counters)



Pictorial



Abstract



It is important that children have a solid understanding of place value for example that one ten is the same as ten ones.

Addition and Subtraction: Year 5

Objectives related to addition and subtraction

Year Five National Curriculum objectives:

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Recall	Mental Calculation Skills	Mental Strategies
•sums and differences of decimals, e.g. 6.5 + 2.7, 7.8 – 1.3 • doubles and halves of decimals, e.g. half of 5.6, double 3.4 • what must be added to any four-digit number to make the next multiple of 1000, e.g. 4087 + □ = 5000 • what must be added to a decimal with units and tenths to make the next whole number, e.g. 7.2 + □ = 8	• add or subtract a pair of two-digit numbers or three-digit multiples of 10, e.g. 38 + 86, 620 – 380, 350+ 360 • add or subtract a near multiple of 10 or 100 to any two-digit or three-digit number, e.g. 235 + 198 • find the difference between near multiples of 100, e.g. 607 – 588, or of 1000, e.g. 6070 – 4087 • add or subtract any pairs of decimal fractions each with units and tenths, e.g. 5.7 + 2.5, 6.3 – 4.8	• count on or back in hundreds, tens, ones and tenths • partition: add hundreds, tens or ones separately, then recombine • subtract by counting up from the smaller to the larger number • add or subtract a multiple of 10 or 100 and adjust • partition: double and adjust • use knowledge of place value and related calculations, e.g. 6.3 – 4.8 using 63 – 48 • partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times)

Addition

Progression of Calculation

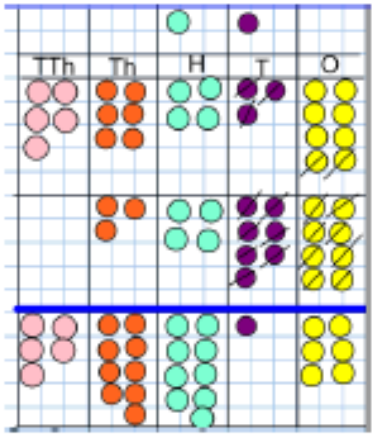
Addition of numbers beyond 1000s

Addition of numbers with up to two decimal places

Addition of numbers with at least 4 digits and decimals with various missing place-holders.

Written methods

56,438 + 3,478 =




```
   1 1
  56438
+  3478
-------
  59916
```

Children extend their use of the compact column method to add numbers with at least four digits and decimals with careful consideration of missing place holders.

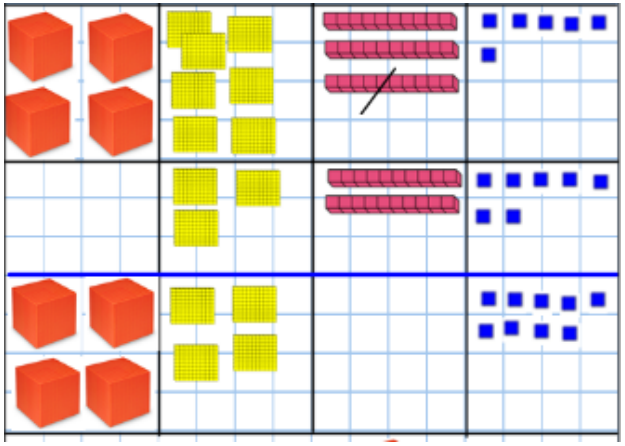
```
  HTO.th
  257.80
+ 492.55
---------
  750.35
  1 1 1
```

Children **should** include zero place-holders to aid layout and understanding of place value.

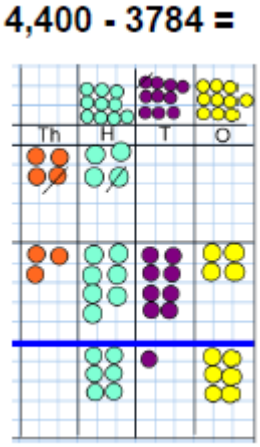
Progression of Calculation

TH H T O – TH H T O
O.t – O.t
O.t h – O.th
O.t h – O.t
O. t – O . t h
Then with larger numbers involving decimals

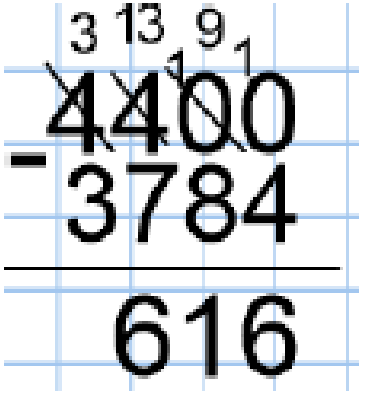
Concrete
(Manipulating Base 10 and counters)



Pictorial



Abstract



The same principle would be used to subtract decimal numbers.

Addition and Subtraction: Year 6

Objectives related to addition and subtraction

Year Six National Curriculum objectives:

- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

Recall	Mental Calculation Skills	Mental Strategies
• sums and differences of decimals, e.g. 6.5 + 2.7, 7.8 – 1.3 • doubles and halves of decimals, e.g. half of 5.6, double 3.4 • what must be added to any four-digit number to make the next multiple of 1000, e.g. 4087 + □ = 5000 • what must be added to a decimal with units and tenths to make the next whole number, e.g. 7.2 + □ = 8	• add or subtract a pair of two-digit numbers or three-digit multiples of 10, e.g. 38 + 86, 620 – 380, 350+ 360 • add or subtract a near multiple of 10 or 100 to any two-digit or three-digit number, e.g. 235 + 198 • find the difference between near multiples of 100, e.g. 607 – 588, or of 1000, e.g. 6070 – 4087 • add or subtract any pairs of decimal fractions each with units and tenths, e.g. 5.7 + 2.5, 6.3 – 4.8	• count on or back in hundreds, tens, ones and tenths • partition: add hundreds, tens or ones separately, then recombine • subtract by counting up from the smaller to the larger number • add or subtract a multiple of 10 or 100 and adjust • partition: double and adjust • use knowledge of place value and related calculations, e.g. 6.3 – 4.8 using 63 – 48 • partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times)

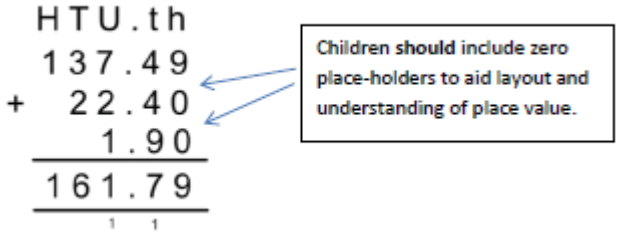
Addition and Subtraction

Progression of Addition Calculation

Addition of numbers with any number of digits
Addition of two or more numbers with at least 4 digits and 3 decimal places
Addition of two or more numbers with at least 4 digits of various sizes and varied decimal places. (e.g. 401.2 + 26.85 + 13)

Written Methods

- Use empty number lines as for Y3, 4 and 5 and continue to recognise when a written or mental strategy (with or without jottings) is more efficient.
- Confident understanding of place value and use compact column method:



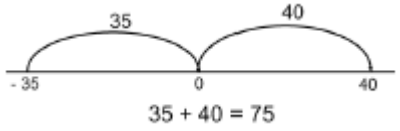
Progression of Addition Calculation

Refine year 5
Difference between 2 negative integers
Difference between positive and negative integers

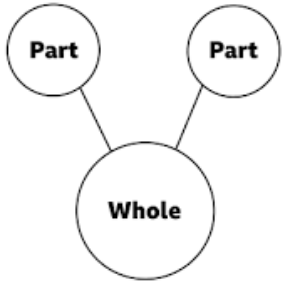
Written methods

- In year 6 children will refine methods taught in year 5 to consolidate the compact methods and work with accuracy and fluency.
- Children will use an empty number line to show differences between negative numbers, and positive and negative numbers.

What is the difference between 40 and -35?



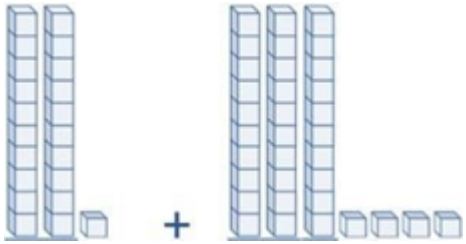
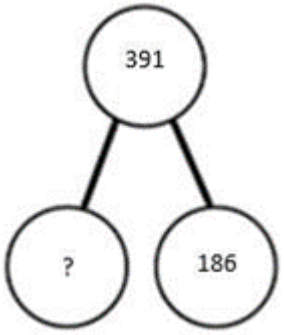
Conceptual Variation addition and subtraction examples:

<table>
<tr><td colspan="4">Different ways to ask 21 + 34</td></tr>
<tr>
<td>
<table>
<tr><td colspan="2">?</td></tr>
<tr><td>21</td><td>34</td></tr>
</table>


</td>
<td>Word problems:

In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

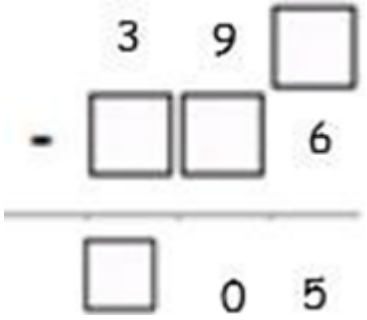
21 + 34 = 55. Prove it</td>
<td>21
+34
____
21 + 34 =
☐
= 21 + 34

Calculate the sum of twenty-one and thirty-four.</td>
<td> 
Missing digit problems:
<table>
<tr><th>10s</th><th>1s</th></tr>
<tr><td>10 10</td><td>1</td></tr>
<tr><td>10 10 10</td><td>?</td></tr>
<tr><td>?</td><td>5</td></tr>
</table>
</td>
</tr>
<tr><td colspan="4">Different ways to ask children to solve 391 - 186</td></tr>
<tr>
<td>


<table>
<tr><td colspan="2">391</td></tr>
<tr><td>186</td><td>?</td></tr>
</table>
</td>
<td>Raj spent £391, Timmy spent £186. How much more did Raj spend?

Calculate the difference between 391 and 186.</td>
<td>☐ = 391 - 186

391
-186
____

What is 186 less than 391?</td>
<td>Missing digit calculations


</td>
</tr>
</table>

Multiplication and Division: EYFS

Objectives related to multiplication and division

- **Solve problems, including doubling, halving and sharing.**

Multiplication

Progression of calculation
Counting in 2s and 10s
Doubling single-digit numbers

Written methods

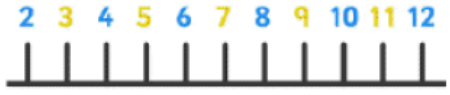
- Children use practical equipment and pictures to create sets of equal amounts and add together by counting.

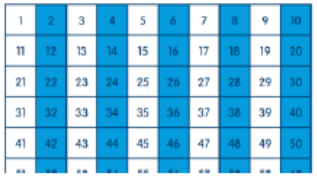


- Children make a record of doubling numbers to 5 using practical equipment, pictures and symbols.

- Through songs, games, real life contexts and using practical equipment, children count in repeated groups of the same size (2s, 10s).

- Children use number lines, number tracks and hundred squares to count in 2s and 10 and notice patterns.





Division

Progression of calculation
Creating equal groups of a set of objects
Sharing a set of objects

Written methods

- Children use practical materials to create equal groups to solve a problem
E.g. How many cars can be made with 8 wheels?

- Through opportunities which arise in the classroom, children share objects equally (this includes sharing into two sets to halve numbers).

- Children will be encouraged to count out objects in 2s and 10s when grouping and sharing if appropriate.

Multiplication and Division: Year 1

Mental skills

Multiplication

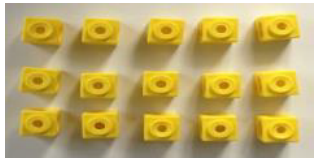
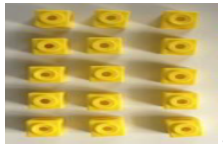
Recognise the size and position of numbers
Count on in different steps 2s, 5s, 10s
Double numbers up to 10
Recognise multiplication as repeated addition
Quick recall of multiplication facts
Use known facts to derive associated facts
Multiplying by 10 and understanding the effect
Multiplying by multiples of 10

Division

Recognise the size and position of numbers
Count back in different steps 2s, 5s, 10s
Halve numbers to 20
Recognise division as repeated subtraction
Quick recall of division facts
Use known facts to derive associated facts
Divide by 10 and understanding the effect
Divide by multiples of 10

Stem sentences

- Times means lots of/sets of
- The total will always (unless multiplying by 0/1) be more than you started with.
- When making an array “lots” go across.


3 lots of 5

5 lots of 3

- Multiplication is the same as repeated addition.
- If you multiply by two the number will always be even.
- If you multiply by five the number will always end in a 0 or a 5.
- If you multiply a number by 10 it will always end in a zero.

Stem sentences

- Division means sharing or grouping
- When dividing or sharing your groups must be equal.
- The total will always (unless dividing by 0 or 1) be less than you started with.
- The first number will always be your number of crosses.
- The second number will always be the amount of circles.
- Division is the same as repeated subtraction.

Objectives related to multiplication and division

Year One National Curriculum objectives

Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

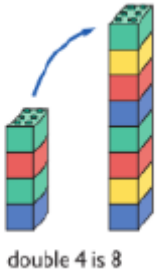
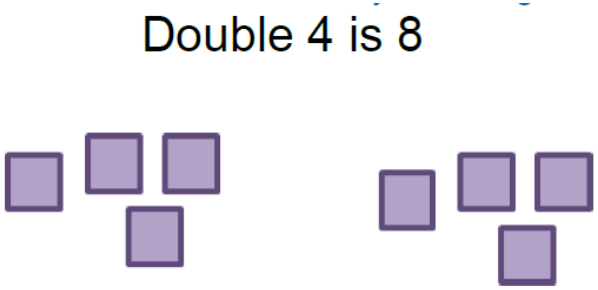
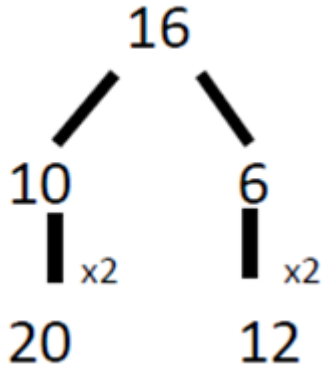
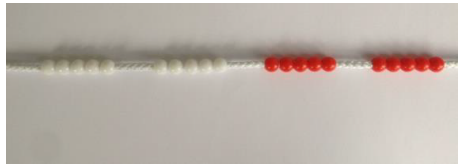
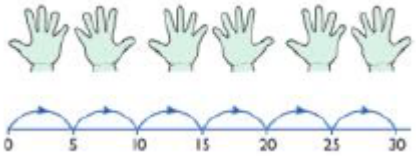
Objectives related to multiplication and division

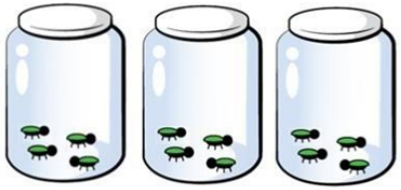
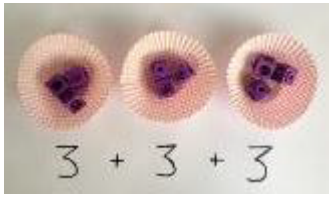
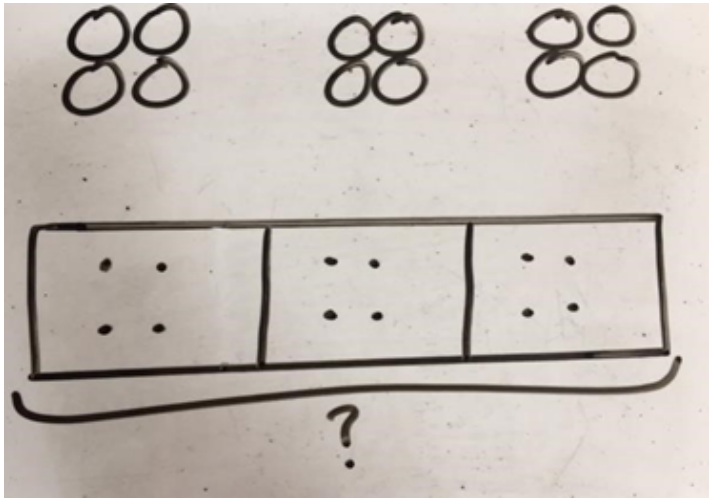
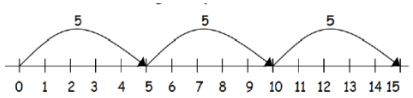
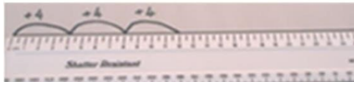
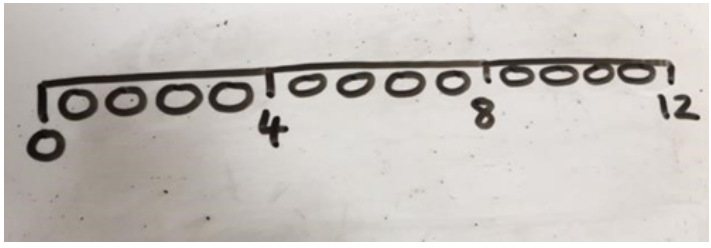
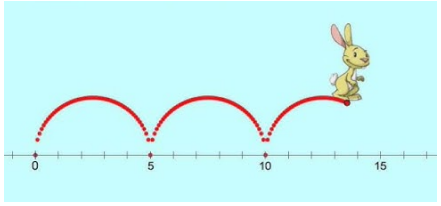
Year Two National Curriculum objectives

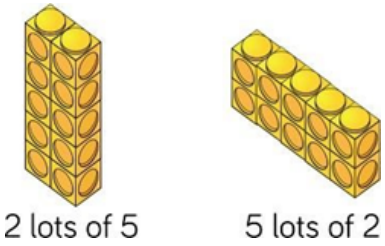
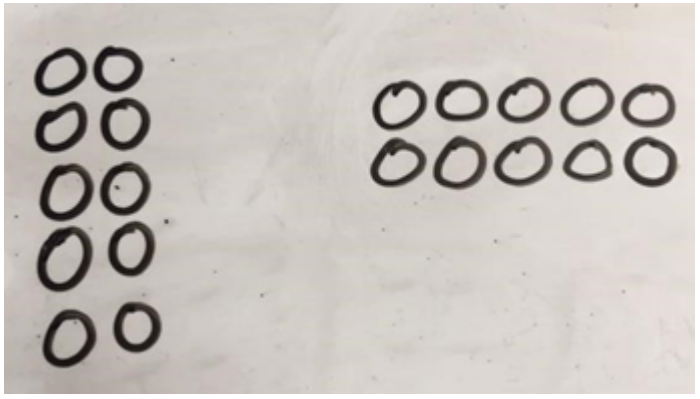
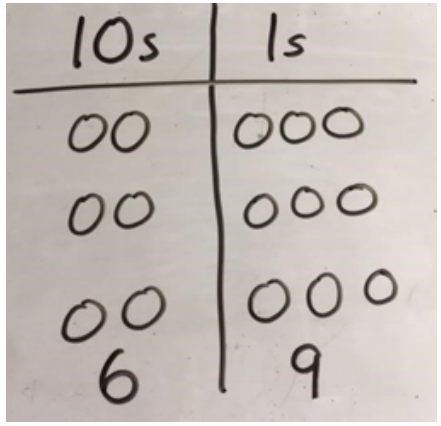
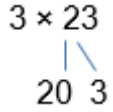
- Recall and use multiplication and division facts for 2, 5, and 10 multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x) and division (÷) and equals (=) signs
- Show that multiplication of two numbers can be done in any order (commutative) and division by one number by another cannot
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Multiplication: Key Stage 1

Progression of calculation
Doubling of numbers to 10
Multiplication of sets of 2s, 5s, and 10

Objective and Strategies	Concrete	Pictorial	Abstract
Solving problems, including doubling.	Use practical activities to show how to double a number.  double 4 is 8	Draw pictures to show how to double a number. 	Partition a number and then double each part before recombining it back together. 
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Repeated addition/grouping	3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group.   	 Children to represent the practical resources in a picture and use a bar model.	Write addition sentences to describe objects and pictures. 3 × 4 = 12 4 + 4 + 4 = 12
Number lines to show repeated groups	   Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.: 	Abstract number line showing three jumps of five. 

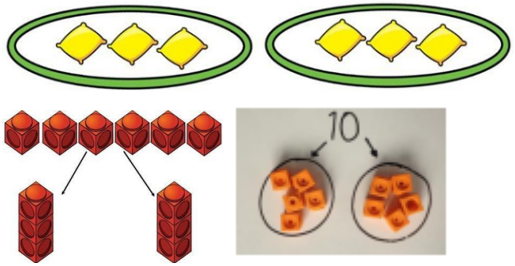
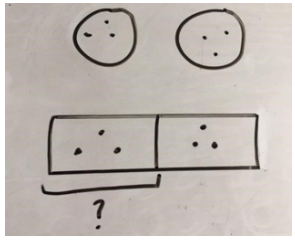
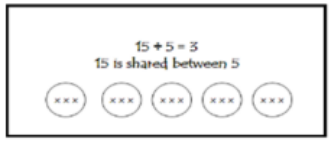
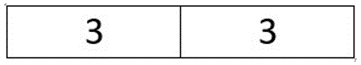
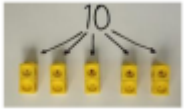
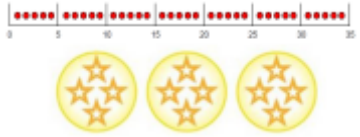
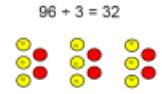
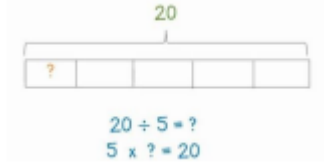
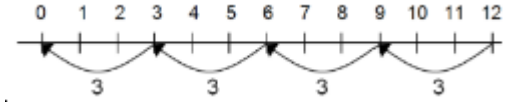
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. 10 = 2 × 5 5 × 2 = 10 2 + 2 + 2 + 2 + 2 = 10 10 = 5 + 5
Formal column method E.g 3 × 23	With place value counters (base 10 can also be used.)  	Children to represent the counters pictorially.  	Children to show 3 × 23 record what it is they are doing understanding.  3 × 20 = 60 3 × 3 = 9 60 + 9 = 69 23 × 3 69

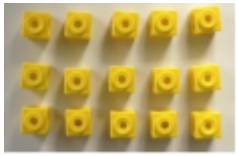
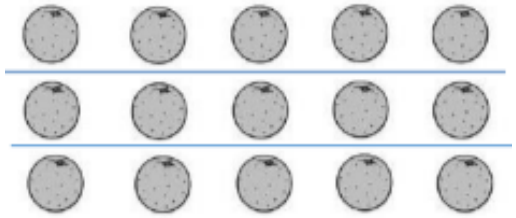
Division: Key Stage 1

Progression of calculation

Doubling of numbers to 10

Multiplication of sets of 2s, 5s, and 10

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to represent sharing pictorially.   6 ÷ 2 = 3	6 ÷ 2 = 3  Share 6 buns between two people.
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   	Use a number line to show jumps in groups. The number of jumps equals the number of groups. Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  	Solving division calculations such as: Five people share 20 cookies. How many do they get each? 20 ÷ 5 = 4

Division within arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created. E.g. 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15 	Draw an array and use lines to split the array into groups to make multiplication and division sentences. 	Find the inverse of multiplication and division sentences by creating four linking number sentences. 2 x 4 = 8 4 x 2 = 8 8 ÷ 4 = 2 8 ÷ 2 = 4

Multiplication and Division: Year 3

Objectives related to multiplication and division

Year Three National Curriculum objectives:

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

Recall	Mental Calculation Skills	Mental Strategies
● multiplication facts for the 2, 3, 4, 5, 6, 8 and 10 times-tables, and corresponding division facts ● doubles of multiples of 10 to 100,e.g. double 90, and corresponding halves	• double any multiple of 5 up to 100, e.g. double 35 • halve any multiple of 10 up to 200, e.g. halve 170 • multiply one-digit or two-digit numbers by 10 or 100, e.g. 7 × 100, 46 × 10, 54 x 100 • find unit fractions of numbers and quantities involving halves, thirds, quarters, fifths and tenths	• partition: when doubling, double the tens and ones separately, then recombine • partition: when halving, halve the tens and ones separately, then recombine • use knowledge that halving and doubling are inverse operations • recognise that finding a unit fraction is equivalent to dividing by the denominator and use knowledge of division facts • recognise that when multiplying by 10 or 100 the digits move one or two places to the left and zero is used as a place holder

Multiplication: Year 3

Progression of Calculation

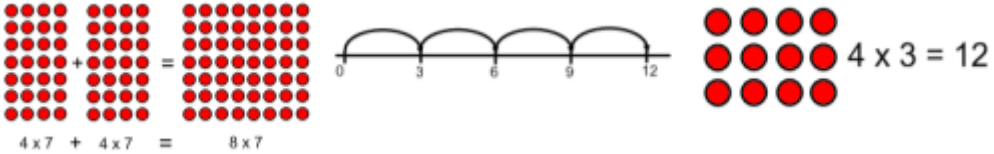
O x (2, 3, 4, 5, 8, 10)

(1-20) x (2, 3, 4, 5, 8, 10) - beyond 20 when confident.

Multiply 1- or 2-digit numbers by 10 and 100

CPA Methods

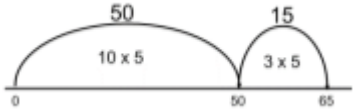
- Children will continue to use the empty number line and/or arrays to reinforce the application of known facts. E.g.



- Arrays will be used to support with learning the associative rule of multiplication and used to learn new facts. e.g.

  8 x 7 = (4 x 7) +(4 x 7) = 28 + 28 = 56

- Children can record using an empty number line.



- When confident, children can record using an informal partitioning method.

  e.g. 28 x 5 = (10 x 5) + (10 x 5) + (8 x 5)

  = 50 + 50 + 40 = 140

  Leading to: 28 x 5 = (20 x 5) + (8 x 5)

  = 100 + 40 = 40

Division: Year 3

Progression of Calculation

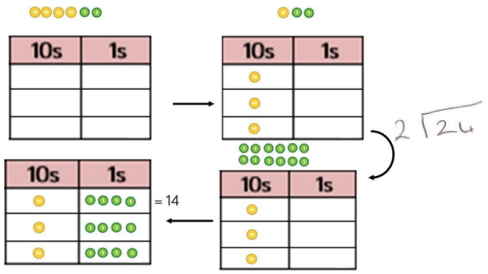
T0 ÷ (2, 3, 4, 5, 8, 10) – answers between 0 and10.

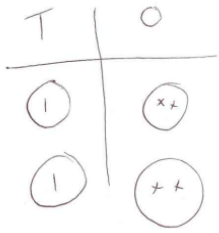
T0 ÷ (2, 3, 4, 5, 8, 10) – answers between 1 and 10 with remainders.

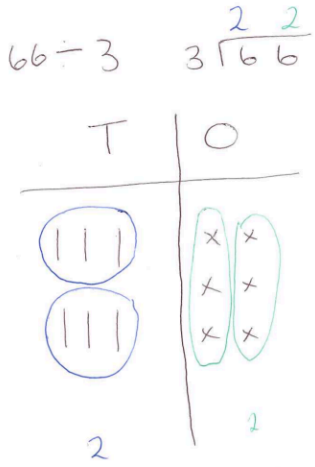
CPA methods

- Division without remainders e.g 42 ÷ 3 = 14

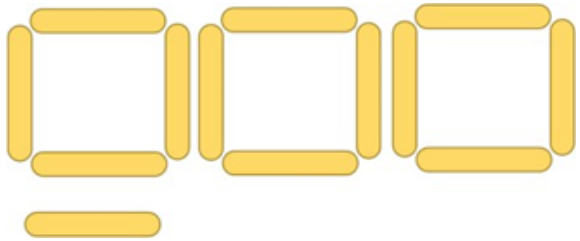
  (Children will be taught the difference between grouping and sharing of amounts).

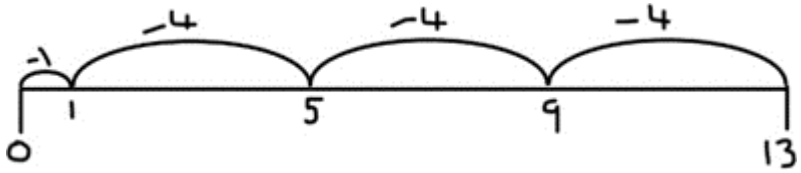




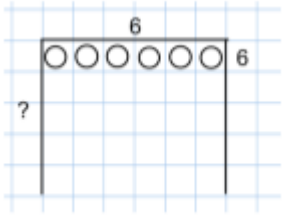


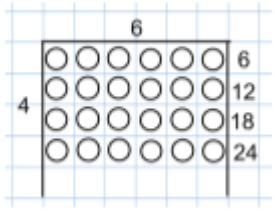
- Division with remainders e.g 13 ÷ 4 = 3 r 1





- Children will use known facts or arrays to represent and answer missing number calculations. E.g. 24 ÷ ? = 6 (*6 equal groups of "something" are in 24*)





Multiplication and Division: Year 4

Objectives related to multiplication and division

Year Four National Curriculum objectives

- recall multiplication and division facts for multiplication tables up to 12 × 12
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Recall	Mental Calculation Skills	Mental Strategies
• multiplication facts to 12 × 12 and the corresponding division facts • doubles of numbers 1 to 100, e.g. double 58, and corresponding halves • doubles of multiples of 10 and 100 and corresponding halves • fraction and decimal equivalents of one-half, quarters, tenths and hundredths, e.g. 310 is 0.3 and 3100 is 0.03 • factor pairs for known multiplication facts	• double any two-digit number, e.g. double 39 • double any multiple of 10 or 100, e.g. double 340, double 800, and halve the corresponding multiples of 10 and 100 • halve any even number to 200 • find unit fractions and simple non-unit fractions of numbers and quantities, e.g. 3/8 of 24 • multiply and divide numbers to 1000 by 10 and then 100 (whole-number answers), e.g. 325 × 10, 42 × 100, 120 ÷ 10, 600 ÷ 100, 850 ÷ 10 • multiply a multiple of 10 to 100 by a single-digit number, e.g. 40 × 3 • multiply numbers to 20 by a single-digit, e.g. 17 × 3 • multiply three digit number by a single-digit, e.g. 123 x 3 • identify the remainder when dividing by 2, 5 or 10 • give the factor pair associated with a multiplication fact, e.g. identify that if 2 x 3 = 6 then 6 has the factor pair 2 and 3	•partition: double or halve the tens and ones separately, then recombine • use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right and zero is used as a place holder • use knowledge of multiplication facts and place value, e.g. 7 x 8 = 56 to find 70 x 8, 7 x 80 = 560 • use partitioning and the distributive law to multiply, e.g. 13 × 4 = (10 + 3) × 4 = (10 × 4) + (3 × 4) = 40 + 12 = 52

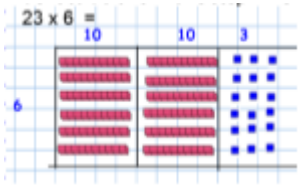
Multiplication: Year 4

Progression of Calculation

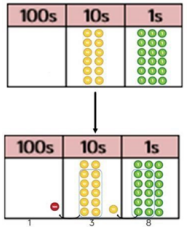
O X TO
O X HTO (expanded column)
O x HTO
Multiplication of numbers by 10, 100, 1000

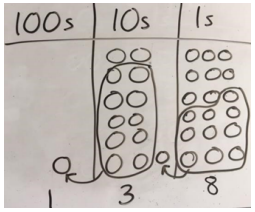
CPA Methods

- Children will continue to use arrays to reinforce the application of known facts. E.g.



- Use place value counters e.g 6 x 23





$$\begin{array}{r} 6 \times 23 = \\ 23 \\ \times\ \ 6 \\ \hline 138 \\ \hline 1\ 1 \end{array}$$

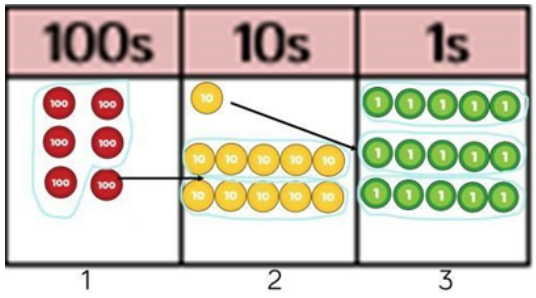
Division: Year 4

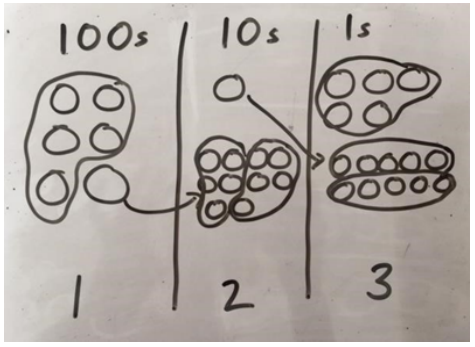
Progression of Calculation

TO ÷ UO = answers 0 – 10
TO ÷ O = answers greater than 10.
As above including remainders.

CPA Methods

- Use place value counters or base 10 before moving to pictorial and abstract representations e.g 615 ÷ 5





$$5\overline{)615} = 123$$

1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Multiplication and Division: Year 5

Objectives related to multiplication and division

Year Five National Curriculum objectives:

- identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally, drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000
- recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})
- solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates

Recall	Mental Calculation Skills	Mental Strategies
• squares to 12 × 12 • division facts corresponding to tables up to 12 × 12, and the related unit fractions, e.g. 7 × 9 = 63 so one-ninth of 63 is 7 and one-seventh of 63 is 9 • Percentage equivalents of one-half, one-quarter, three-quarters, tenths and hundredths	• multiply and divide two-digit numbers by 4 or 8, e.g. 26 × 4, 96 ÷ 8 • multiply two-digit numbers by 5 or 20, e.g. 320 × 5, 14 × 20 • multiply by 25 or 50, e.g. 48 × 25, 32 × 50 • double three-digit multiples of 10 to 500, e.g. 380 × 2, and find the corresponding halves, e.g. 760 ÷ 2 • find the remainder after dividing a two-digit number	• multiply or divide by 4 or 8 by repeated doubling or halving • form an equivalent calculation, e.g. to multiply by 5, multiply by 10, then halve; to multiply by 20, double, then multiply by 10 • use knowledge of doubles/halves and understanding of place value, e.g. when multiplying by 50 multiply by

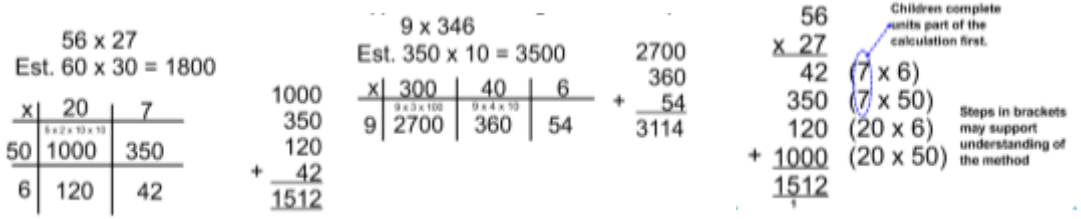
• factor pairs to 100 • prime numbers to 19	by a single- digit number, e.g. 27 ÷ 4 = 6 R 3 • multiply and divide whole numbers and decimals by 10, 100 or 1000, e.g. 4.3 × 10, 0.75 × 100, 25 ÷ 10, 673 ÷ 100, 74 ÷ 100 • multiply pairs of multiples of 10, e.g. 60 × 30, and a multiple of 100 by a single digit number, e.g. 900 × 8 • divide a multiple of 10 by a single-digit number (whole number answers) e.g. 80 ÷ 4, 270 ÷ 3 • find fractions of whole numbers or quantities, e.g. 2/3 of 27, 4/5 of 70 kg • find 50%, 25% or 10% of whole numbers or quantities, e.g. 25% of 20 kg, 10% of £80 • find factor pairs for numbers to 100, e.g. 30 has the factor pairs 1 × 30, 2 × 15, 3 × 10 and 5 × 6	100 and divide by 2 • use knowledge of division facts, e.g. when carrying out a division to find a remainder • use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right relative to the decimal point, and zero is used as a place holder • use knowledge of multiplication and division facts and understanding of place value, e.g. when calculating with multiples of 10 • use knowledge of equivalence between fractions and percentages, e.g. to find 50%, 25% and 10% • use knowledge of multiplication and division facts to find factor pairs

Multiplication: Year 5

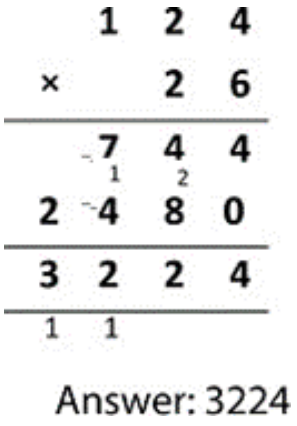
Progression of Calculation
HTO x O
ThHTO x O
TO x TO
O x O.t

CPA Methods

- Informal methods such as the grid method can be used to support understanding before expecting pupils to record using the expanded column method



- When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract:



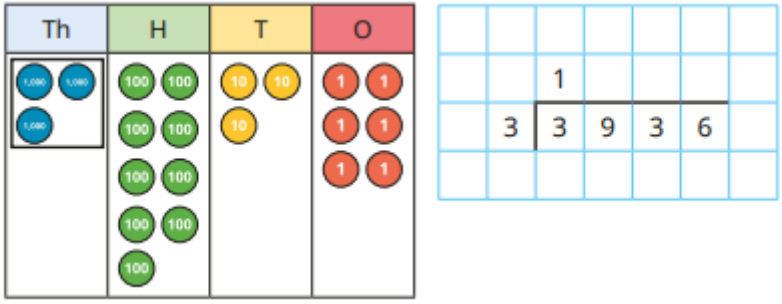
Answer: 3224

Division: Year 5

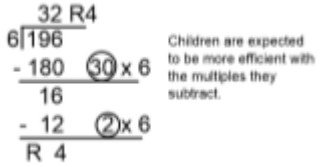
Progression of Calculation
TO ÷ O
HTO ÷ O = TO
HTO ÷ O = HTO
THTO ÷ O =
As above with remainders.

CPA Methods

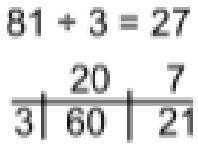
- Concrete and pictorial representations should link to PV counters to aid understanding



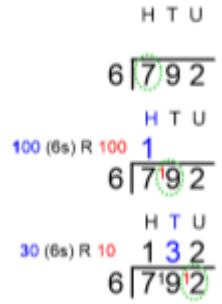
- Informal methods such as chunking (vertical repeated subtraction) or partitioning methods (depending on mental subtraction skills) can be used to divide larger numbers with increasing efficiency.



Children are expected to be more efficient with the multiples they subtract.



- The children will use an expanded method of the formal algorithm before the teacher demonstrates the short method. E.g. 792 ÷ 6 = 132



Multiplication and Division: Year 6

Objectives related to multiplication and division

Year Six National Curriculum objectives:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the 4 operations
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

Recall	Mental Calculation Skills	Mental Strategies
• squares to 12 × 12 • squares of the corresponding multiples of 10 • prime numbers less than 100 • recall common multiples, factors and prime numbers of 2 digit numbers • equivalent fractions, decimals and percentages for hundredths, e.g. 35% is equivalent to 0.35 or 35/100	• multiply pairs of two-digit and single-digit numbers, e.g. 28 × 3 • divide a two-digit number by a single-digit number, e.g. 68 ÷ 4 • divide by 25 or 50, e.g. 480 ÷ 25, 3200 ÷ 50 • double decimals with units and tenths, e.g. double 7.6, and find the corresponding halves, e.g. half of 15.2 • multiply pairs of multiples of 10 and 100, e.g. 50 × 30, 600 × 20 • divide multiples of 100 by a multiple of 10 or 100 (whole number answers), e.g. 600 ÷ 20, 800 ÷ 400, 2100 ÷ 300 • multiply and divide two-digit decimals such as 0.8 × 7, 4.8 ÷ 6	• partition: use partitioning and the distributive law to divide tens and ones separately, e.g. 92 ÷ 4 = (80 + 12) ÷ 4 = 20 + 3 = 23 • form an equivalent calculation, e.g. to divide by 25, divide by 100, then multiply by 4; to divide by 50, divide by 100, then double • use knowledge of the equivalence between fractions and percentages and the relationship between fractions and division • recognise how to scale up or down using multiplication and division, e.g. if three oranges cost 24p: one orange costs 24 ÷ 3 = 8p four oranges cost 8 × 4 = 32p • Use knowledge of multiplication and division facts to

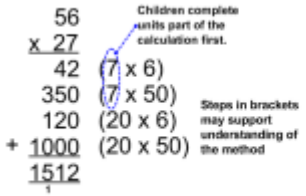
	• find 10% or multiples of 10%, of whole numbers and quantities, e.g. 30% of 50 ml, 40% of £30, 70% of 200 g • simplify fractions by cancelling • scale up and down using known facts, e.g. given that three oranges cost 24p, find the cost of four oranges • identify numbers with odd and even numbers of factors and no factor pairs other than 1 and themselves	identify factor pairs and numbers with only two factors

Multiplication: Year 6

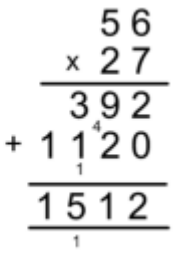
Progression of Calculation
Th H T O x O
H T O x T O
Th H T O x T O
O x O. t h

CPA Methods

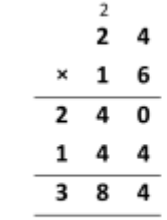
- Children will move on from setting out their partial products in the expanded column method to the compact method of multiplication.



When confident, children are to use the formal algorithm.

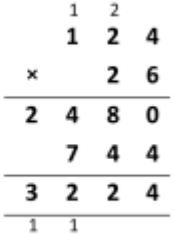


24 × 16 becomes



Answer: 384

124 × 26 becomes



Answer: 3224

Division: Year 6

Progression of Calculation

THTO ÷ O (with decimal and fraction remainders**, depending on context).**

HTO ÷ TO

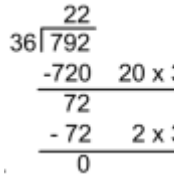
THTO ÷ TO

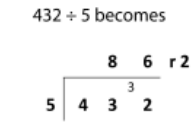
O.t ÷ O

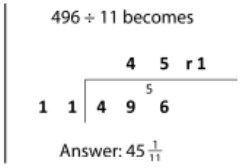
TO.t ÷ O

CPA Methods

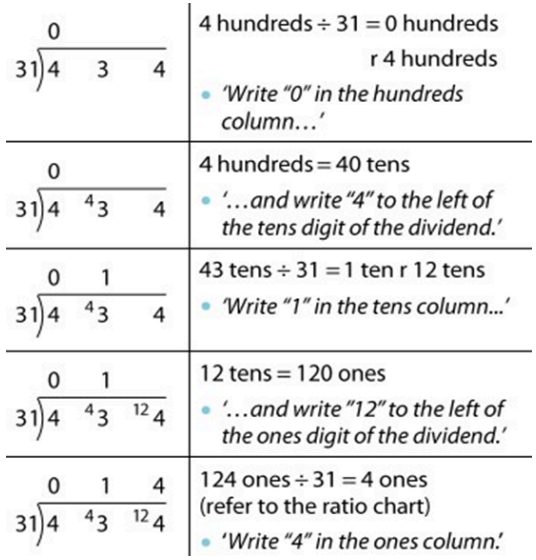
- Year 5 formal and informal methods for short division will be consolidated. Additionally, remainders will be expressed either as a fraction of the divisor or as a decimal. Children will be taught both chunking and the short method of division.

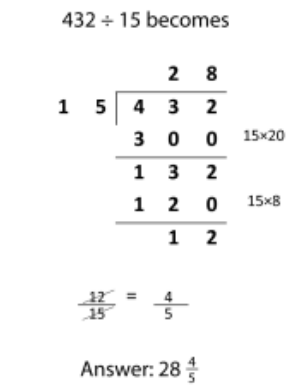


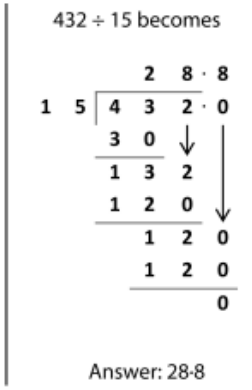


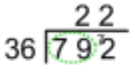


- • If confident, children use the formal algorithm.

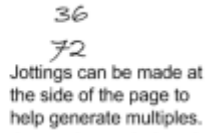






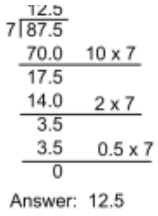


Circling the digits that are being worked with can help with partitioning the number.



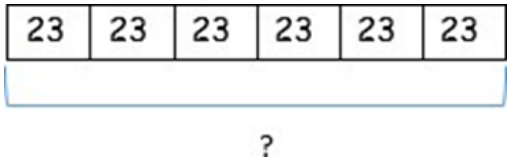
Jottings can be made at the side of the page to help generate multiples.

- Children use chunking to divide numbers with one decimal place by a single digit number.



Children in Year 6 will be encouraged to choose a method suited to them.

Conceptual Variation multiplication and division examples:

<table>
<tr><td colspan="4">Different ways to ask children to solve 6 × 23</td></tr>
<tr>
<td> 
</td>
<td>Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138</td>
<td>Find the product of 6 and 23

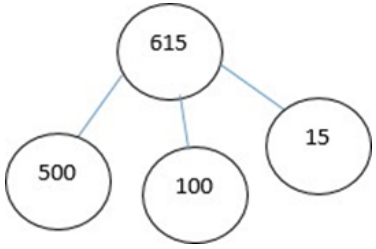
6 × 23 =

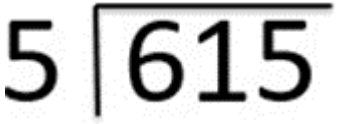
□ = 6 × 23

6 × 23 = ___

23 × 6 = ___</td>
<td>What is the calculation? What is the product?

 
</td>
</tr>
<tr><td colspan="4">Different ways to ask children to solve 615 ÷ 5</td></tr>
<tr>
<td>Using the part whole model below, how can you divide 615 by 5 without using short division?

 
</td>
<td>I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?</td>
<td> 

615 ÷ 5 =

[ ] = 615 ÷ 5</td>
<td>What is the calculation?
What is the answer?

 
</td>
</tr>
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